THE STREHL RATIO AND WHAT IT MEANS

by Dick Suiter

You probably have heard of something called the *Strehl ratio* as a method of specifying telescope quality. You may have seen it mentioned on the Internet, where a small cadre of folks claim that it completely supersedes all other measures, or in *Sky & Telescope*, where it is breaking into the mainstream use in advertisements. Is it so superior to other one-number criteria, like encircled energy in the Airy disk or a peak-to-valley (P-V) limit? Is it the latest, greatest, thing?

The Strehl ratio was named after a Karl Strehl who in 1902 drew special attention to this form of specification in the journal *Zeitschrift für Instrumentenkunde*. However, it was first calculated for 1/4 wavelength P-V spherical aberration by Rayleigh himself in the same paper he used to set forth the 1/4 wave limit (*Philosophical Magazine*, Series 5, Vol 8(50), pp 403-411, November 1879). A later optical worker, André Maréchal, found a close approximation to the Strehl ratio for small aberrations based on the root-mean-square (RMS) deviation of the wavefront surface. Now that there was a convenient way of calculating it, the use of the Strehl ratio took off.

What is the Strehl ratio? **It is simply the ratio of the highest intensity in the aberrated image to the highest intensity of a perfect aperture the same size.** This apparently straightforward definition has some pitfalls, however. First of all, no perfect aperture exists that you can compare the image to. You have to infer the Strehl ratio from other information. Second, it is based on an infinitesimal limit, rather than a finite sample size, so if a perfect or at least a comparison aperture existed, you still couldn't directly measure it. Third, you are faced with picking apart the intensity loss due to simple transmission loss and intensity loss due to diffraction pattern modifications. It is only the latter that you are interested in.

If we base our quality criterion on effects purely having to do with transmission, we are forced to admit nonsensical results. For example, say you had a large, perfect, circular aperture. The Strehl ratio is 1. You look at the Moon and other than being dazzling, it looks fine. You put in a perfect neutral density filter that reduces the transmission to 10 percent, and the darker image is improved immensely. Yet, if your Strehl ratio were based on transmission, you would have a Strehl ratio of 0.1! There are even more ridiculous cases. A solar filter might remove 99.99% of the light, but we don't say it has a Strehl of 0.0001.[1]

Now, no one would seriously say that the telescope with simple filters have diminished Strehls, because that is too obvious, but there is a case where even optically-sophisticated people are fooled. The uncorrected diffraction calculation for a perfect 30% obstructed aperture yields 0.83 at the center. Since this matches the anecdotally determined rough equivalence of 1/4 wave spherical aberration (Strehl =0.8), people assume that they have calculated it correctly. Yet if you add up all the energy in the image, you get 0.91 of the clear aperture case. In other words, the area of the clear annulus is $(1)^2 - (0.3)^2 = 0.91$. Because you have an obstruction you have placed a very mild filter in front of the lens. The light that hits the rear of the obstruction -- no doubt with a dull thud -- should not have anything to do with the diffraction pattern. It is only the light that passes by that contributes.

For our pure diffraction calculation, we should renormalize (i.e., make equal to a certain value) the energy of both the test aperture and the theoretical aperture and divide them. What you are left with is the diminishment due to diffraction alone. This corrected Strehl ratio for the otherwise perfect 30% apertures is 0.91 again! The apparent doubling of the energy loss at the center of the diffraction patter due to

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[1] There is a common usage for something called the *system Strehl ratio* by the adaptive optics crowd, but that has little to do with the classical definition of Strehl ratio. It depends on gathering enough photons to sense the aberrations caused by the atmosphere and depends again on transmission. It could well have been called any one of a number of things. I would prefer "ensquared energy ratio" since the sensor array is usually divided into squares.

obstruction was called the "extinction paradox" by H.C. van de Hulst in his classic book *Light Scattering by Small Particles*. There he used the interesting analogy of a flowerpot sitting on a window sill. If you add up all the energy close up you find it is proportional to the area of the window minus the energy of the pot. If you measure the intensity far away at the center of the diffraction pattern it is about proportional to twice the area of the pot -- half is diffraction and half is simple shading. If you are describing diffraction, you should remove the effect of the simple shading.

I have pointed out that this generalized Strehl ratio is the same thing as the encircled energy ratio at the center of the pattern,[2] or EER(r→0). It is also the same thing as the 2-dimensional integral of the normalized modulation transfer function (MTF). It is this functional description that is recognized as the most complete description of optical quality. It is my personal opinion that the either EER(r→0) or the integral under the normalized MTF curve should become the new definition of the Strehl ratio. Then none of this confusion could arise.

Anyway, since the Strehl ratio has been renormalized to 0.91 for a 30% obstructed aperture, does that mean that we shouldn't view a 30% obstruction as much *better* than 1/4 wave of spherical aberration? No, it does not. If you take a different encircled-energy criterion, the encircled energy at the edge of the Airy disk (that is, EER(r=1.22)) it droops to 0.82 again, but this time it has already been renormalized.

We seem to have shown that the Strehl ratio has been soundly trounced by another quality criterion. But the roles could have been reversed if another case were examined. What we should take from this is that *no single-number criterion is the final word in describing optical quality*. Take the comments of those that claim that it supersedes all other criteria as suspect.

The Strehl Ratio II

To recap the argument last month, we saw how the normalized Strehl ratio (that is, the Strehl ratio based only on the shape of the image rather than the overall brightness) is a little weaker predictor of optical quality in the case of obstruction. According to this normalized ratio, the common threshold of optical quality (Strehl = 0.8) is not passed until the obstruction reaches over 45%. Theoretical blurrings of high-

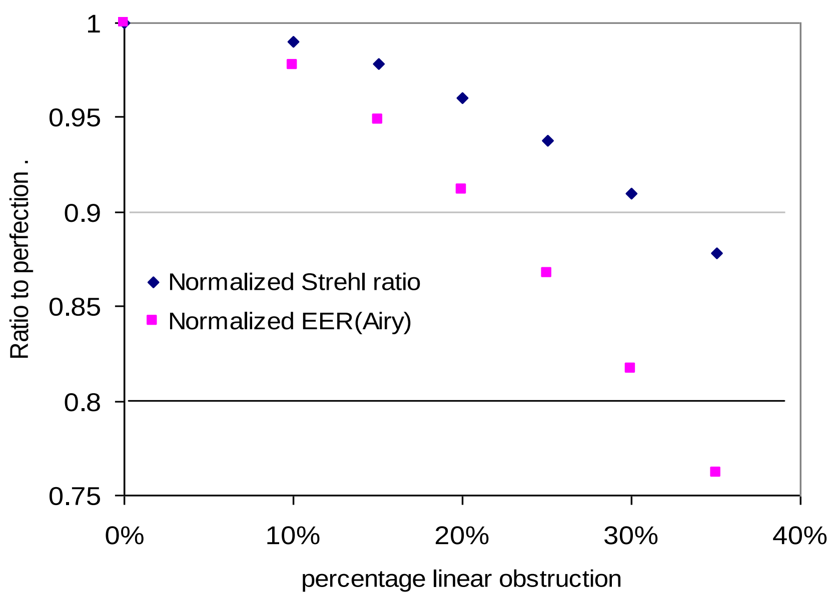

Figure 1. EER at the edge of the Airy disk is a better predictor than EER at the center of the image (i.e., Strehl)

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[2] Suiter, *Star Testing*, pg 197.

quality planetary images with the shape of the diffraction disk reveal that the actual image degrades much faster than this. The origin of the 80% Strehl was a calculation for 1/4 wavelength of spherical aberration at best focus. Blurring done with 30 to 33% obstruction and 1/4 wavelength of spherical aberration are witnessed to be almost identical.

Indeed, we found that the criterion which gives a much better account of itself is not the Strehl ratio but the integrated energy ratio to the edge of the Airy disk. We can see in the diagram below that the normalized energy integrated to the edge of the Airy disk crosses 80% at 32%, neatly matching experience.

But there are some uses for which Strehl ratio is more carefully tuned, such as its use to describe the degradation of aberrations. It was noticed to cross 80% for 1/4 wave of low-order spherical aberration. What about other aberrations? The coma induced by tilting a mirror crosses a Strehl ratio of 80% at a peak-to-valley of about 1/3 wavelength if the correct center position of the image is chosen. For astigmatism, the crossing point is nearer 3/8 wavelength for roundest focus. For the next highest order of spherical aberration (the one containing one more "wiggle"), it is 1/4 wavelength at minimum peak-to-valley wavefront error again (see Figure 2). The peak-to-valley representation seems to wander around a good deal for the same Strehl ratio. The Strehl ratio, because it is based on the weighted average whole wavefront, is more stable.

A special case will demonstrate this: a small (0.2 mm) pit that survives polishing and is aluminized is supposedly a part of an otherwise perfect 8-in mirror, yet it diffracts only a millionth of the light hitting the mirror. Therefore, the Strehl ratio due to this lone error is 0.99999. However, its diameter is 0.1 mm ~ 100 μm ~ 200 wavelengths of light, making its peak-to-vally number 400 wavelengths error! Only because peak-to-valley measures are not inferred from microscopic determinations does its use not ruin evaluation. People don't infer wavefront from tiny spots but large zones.

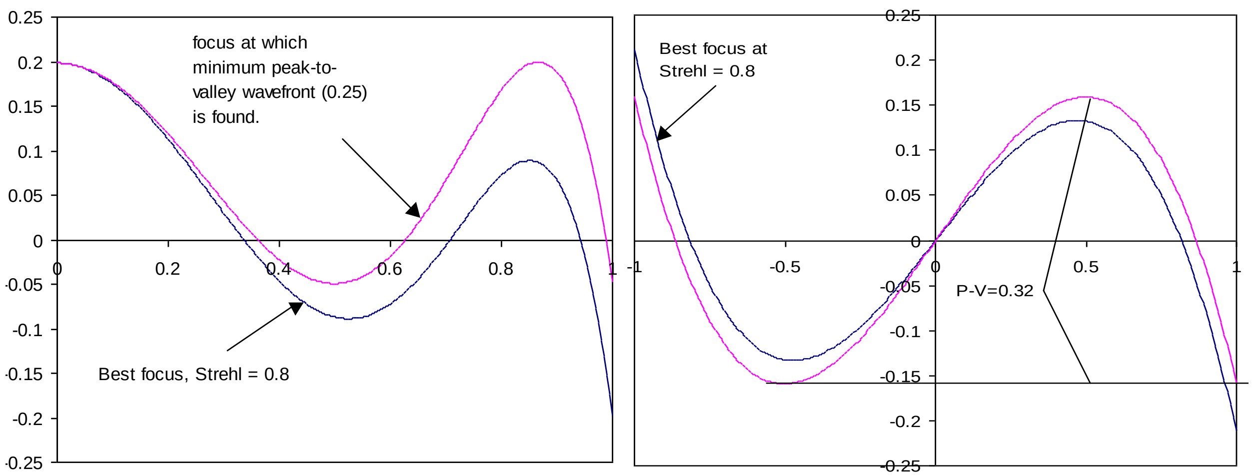

Figure 2. The focus and tilt settings to derive maximum Strehl and minimum peak-to-valley are shown for high-order spherical aberration and coma wavefronts. In both, minimum Strehl = 0.8.

Lets look at the advantages of the Strehl ratio. First of all, it is really easy to compute. There are three really big concepts in optics and they're all related. I call them the "grand triad." They are diagramed in Figure 3. Before I say anything else, let me say that if this is your first contact with these concepts, don't expect to understand everything. Just take from this diagram that we can reach the Strehl ratio from a number of paths.

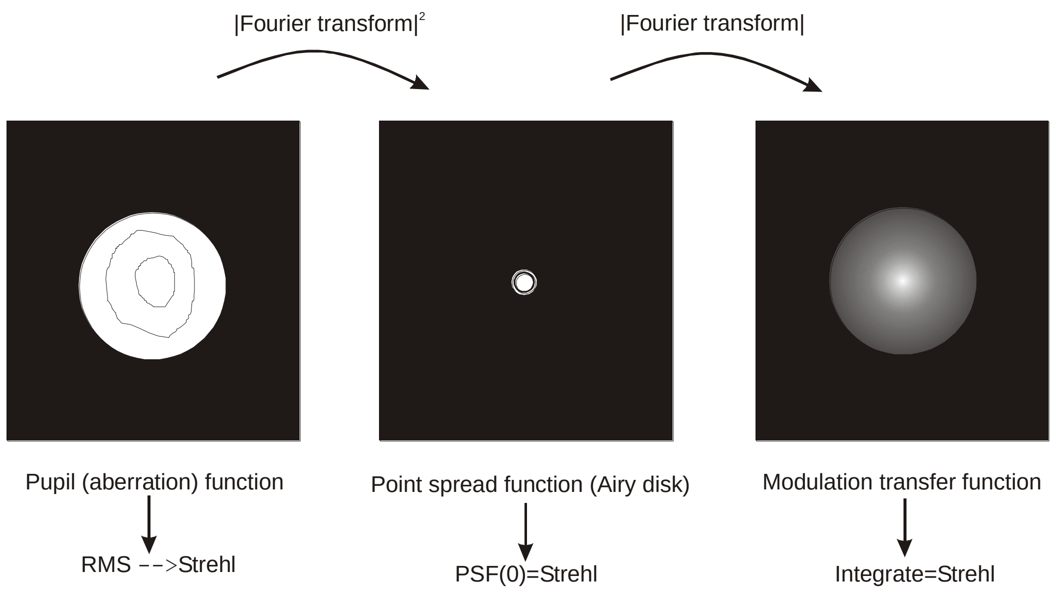

Figure 3. The "grand triad" of optics: left, the pupil function; middle, the point spread function; and right, the modulation transfer function.

First, if we take the standard deviation (called RMS, or root mean square) of the best focus on the wavefront, and fit it through a little mathematical formula, we derive the Strehl ratio. This is the way that most people do it. Taking the mathematical operation known as the Fourier transform, and squaring it, yields the point-spread function (PSF), or just the appearance of a star. Normalizing the PSF properly and taking the maximum value (usually at the center) gives the Strehl ratio again. Practically no one does it this way from real data because the normalization is tough. Taking yet another Fourier transform and deriving its magnitude yields the 2-dimensional *modulation transfer function* or the ability to preserve the contrast of finer and yet finer bars. MTF is easy to normalize for all but wide scattering. Calculating the volume under the normalized MTF is called integration, and it again yields the Strehl ratio. This calculation could be a more uniform definition of the Strehl ratio. Things like the extinction paradox (discussed last month) are automatically compensated for. The Strehl ratio has wormed its way right into the core of modern optics.

Still we should not accept this criterion without some critical review. Strehl ratio is proportional to the brightness of the diffraction pattern at its center, and it can be shown to be equal to the encircled energy ratio at the center. There are many values of these encircled energy ratios (indeed, the collection of all of them can be shown to be related to the MTF). Who is to say that the best predictor of performance is the one in the center? There are a number of contenders, of which the encircled energy ratio calculated at the edge of the Airy disk is the chief. We could also choose the encircled energy at some small radius representing the seeing limit (like 1/3 arcsecond) which may make more sense for really big instruments. Another alternative is the encircled energy at the halfway-down point of the Airy disk. My personal favorite is the encircled energy ratio to 1, where the Airy side is 1.22. This is just the inverse of the maximum spatial frequency of the MTF. Any one of these criteria may, given the application, be better, and yet people tout the Strehl ratio almost to the fanatical level, with bits of foam at the corners of their mouths and a wild look in their eyes.

In a way, you can understand it. If all things were equal, and if all test measurements were given as a set of tightly spaced wavefront amplitudes all over the pupil, I could get behind the movement to convert from peak-to-valley criteria to Strehl-ratio or some other encircled-energy ratio criterion. But so often, people are talking about the same 4 to 6-zone Couder-mask Foucault test across the diameter. To say of such a test that Strehl ratio is vastly superior to a peak-to-valley criterion is to stretch credibility. For one thing, the first three aberrations measured by such a test -- defocus, low-order spherical aberration, and higher-order spherical aberration -- *all* cross a Strehl ratio of 0.8 when they can be refocused to 1/4-wave aberration. There is only a scale factor between the two tests. Many aberrations, such as odd-order or astigmatic terms are not measured well in the Foucault test as many amateurs conduct it. You won't measure them using this technique in the Strehl calculation either.

So, the Strehl ratio is just one of many encircled energy criteria. It is better than peak-to-valley in principle, but the way most amateurs conduct testing, it offers no obvious benefit.