# REDUCING SPHERICAL ABERRATION IN REFLECTORS

(original April 2004) by Dick Suiter

This is going to be a series of articles explaining the differences between simple Newtonian reflectors and catadioptrics (mixed lens/mirror telescopes). Many people buy and use "cats" without knowing exactly what they can do and what they cannot, and in general ascribe miraculous performance to them.

SPHERICAL ABERRATION

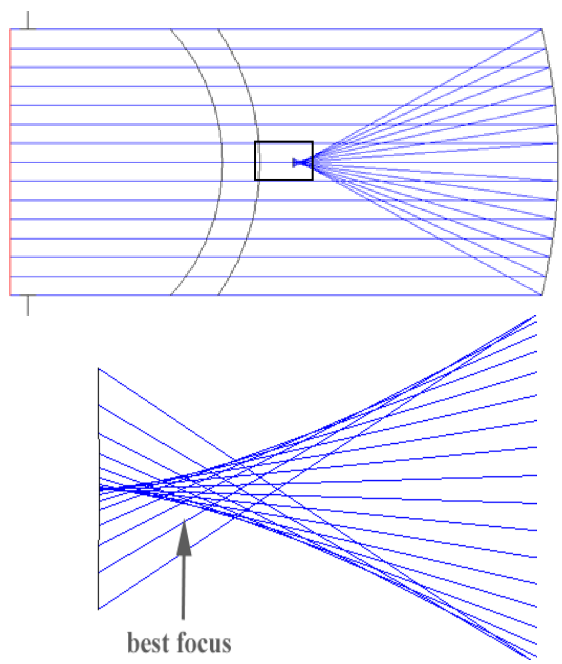


Figure 1. Rays as reflected by a sphere. The other correctors are made of air in this example, and hence don't contribute.

A perfect optical system takes a set of rays (or flat wavefront) from a distant object and directs the rays toward a single central point (or converts that flat wavefront to a spherical converging one). After the optical system has finished processing the light, if it directed at more than one point (or if the converging wavefront is not spherical), it is said to possess "aberrations." All of the aberrations discussed here will be more precisely called "residual aberrations" because they are what is left over after you subtract the perfect conditions from them.

A spherical mirror might be the first guess you make for the ideal shape of your optical surface, because a spherically-converging wavefront is your goal, but you would be wrong. A sphere will return light rays perfectly only for sources near its center, but distant objects are far from its center. Figure 1 shows the blur near focus of a distant source that has reflected from a sphere. The resulting cone-shape structure looks like a horn, **with light rays from the edge crossing short of the central focus**. Aberrations can be of any form, and they all deviate from a sphere, but this aberration has the special name "spherical aberration" (or SA) because it is the same at every angle around the optical surface and has no azimuthal axis of symmetry like coma and astigmatism.

*A little aberration terminology*

It depends on how we want to describe spherical aberration. Camera designers, who seldom deal with diffraction-limited optics, generally write it in the ray-optics form of transverse displacement from the focal point, where this displacement is a power series in radius of the lens (don't worry if you don't understand the equations; this is just terminology):

$$T = ar + br^3 + cr^5 + \dots$$

The coefficient $a$ is the amount of defocus, $b$ is the amount of "third-order spherical aberration," and $c$ is the amount of "fifth-order spherical aberration." It has to be all odd powers because diversion to the other side of the focus must be negative. This description originated with a 19th century optical designer named Seidel, and has hung around ever since.

When you are describing diffraction-limited telescopes, in my opinion, you can derive more useful results by writing the spherical aberrations in terms of the wavefront deviation from a perfect converging sphere:

$$W = P + Ar^2 + Br^4 + Cr^6 ...$$

Here, the coefficient related to defocus in the previous equation is $A$, what formerly was 3rd-order SA is $B$ and multiplies the fourth power of the radius $r^4$, and 5th order SA multiplies the sixth-order term of the wavefront and is $C$. This time, they multiply even powers of $r$ because the wavefront has to be the same on either side of the mirror. $P$ is a coefficient that stands for "piston" and is used only to mathematically center the wavefront on the ideal converging sphere. Indeed, one can derive $T$ from $W$ by taking the derivative and scaling properly. Again, I'm just explaining terminology. Don't worry if you don't understand the math.

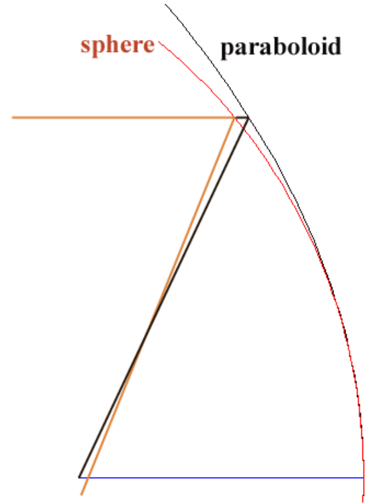


Figure 2. Using a paraboloidal mirror to fix spherical aberration

*So, how do you fix aberrations?*

First, the Newtonian telescope fixes all orders of spherical aberration by replacing a spherical mirror with a paraboloidal. The result appears in Figure 2. Note that rays from nearer the edge hit "late" but are directed at a milder angle that brings them all to the same focus at the end of the blue line. All the rays are depicted near focus in Figure 3.

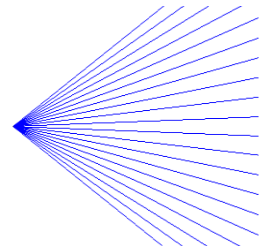

Figure 3. The rays of Figure 1 corrected by a paraboloid

Also notice that the paraboloid of Figure 2 is flatter on the edge that the middle. Anyone who has made a mirror knows that you deepen the center of a paraboloid, not flatten the edge. This procedure, however, is a fabrication trick only. When you deepen a telescope mirror to a paraboloid, you are selecting a reference sphere from a different family of curves with a slightly shorter focal length.

*Catadioptrics*

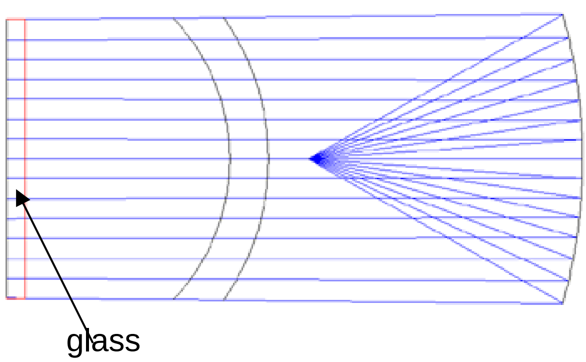


Figure 4. Schmidt spherical aberration correction

Changing the curve of the mirror to the parabolic form, however, drives mirror-making to somewhat of an art form. Manufacturers don't like it, because it requires skilled workers. They like workers who won't disrupt production if they quit or, even worse, go next door and set up shop as competitors.

More to the point, a single optical element has only one configuration. If you want to focus to infinity, you need the paraboloid of the Newtonian. Off-axis performance suffers if the paraboloid has a low ratio of focal length to aperture. Below f/4 or 5, the diffraction-limited angle in telescopes of moderate size is small because of the introduction of off-axis aberrations such as astigmatism and coma. A 10-inch f/4.5 telescope has a diffraction-limited angle of only 3 arcminutes, or only 1/12th inch at the focal plane. A slight tweak of the alignment screws is enough to place this outside the reach of a high-power eyepiece. Perhaps by removing the correction of spherical aberration from the focusing optic and putting it in a low- or zero-power corrector located elsewhere, you can achieve better off-axis performance. There are many ways of doing this, but the most common are Schmidt systems and Maksutov systems.

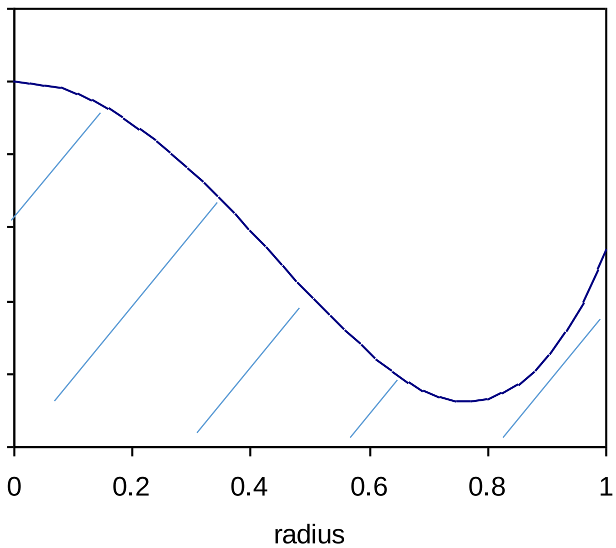

Figure 5. Surface shape from the center outward.

*The first solution: Schmidt optics*

The first solution to the problem of using corrector plates was found by Bernhard Schmidt in the 1930s. It consisted of placing the aperture stop at the center of curvature of the spherical primary and putting a glass plate upon which a special curve has been worked in the aperture. If you think about what is needed, it is not a simple weak lens. A lens would just act like a pair of eyeglasses in refocusing the image. Looking at the above expression for wavefront shape, we see that the lowest non-trivial term is the $Br^4$ term. If the shape of the curve counteracted the coefficient $B$ inherent in the spherical primary with a strength $-B$, then the lowest-order (and strongest) term of spherical aberration would be $Cr^6$. You get the correction appearing in Figure 4. The active optical element is the thin one at the front (the circular Maksutov arcs are still filled with air).

The trouble is, this 4th-order curve is complicated. Figure 5 shows the shape of a radial cross-section. If the curve is to the front, the glass is the region at the bottom. The plate is thickest at the center, dips to a minimum toward the outer side, and then rises to be a little thicker right at the edge. Such a "dipsy-doodle" is contrary to the spirit of fast spherical “cave-man” optics, but the way out of that problem will be explained below. The amplitudes are very slight, only a fraction of a millimeter for the roughly 6-inch f/1 Schmidt camera of the example. For a typical slow Schmidt-Cassegrain, the amount of glass removed is so small that the corrector plate looks flat.

How is such a shape made? The way that Schmidt makers reduce the manufacture of such a strange shape to repeatable practice is by grinding an inverse curve into expensive conformal vacuum plate holders. The optician places a good parallel-plate blank on the holder and sucks it down with a vacuum. The upper surface is then ground and polished flat or to a long radius sphere and the vacuum released. The resulting plate pops out with the required`1 curve. You don't need a skilled artisan to make anything except the holder, just a careful operator when using it. This is something that is cost-effective if you are going to make hundreds if not thousands. Amateurs have been successful in making a few Schmidt optics by hand, but usually regard the effort as an inefficient use of time. This is an activity that responds to economies of scale.

*The second solution: Maksutov optics*

The other solution is to leave the flat corrector at the front filled with air and fill the curved corrector with glass. This resembles a flat parallel-plate at the center and drives the edge rays farther out than they would have been when undiverted. Go back to Figure 2 and puzzle out what’s going on. These diverted rays are dragged into alignment with the central rays by main force. The difficult thing is to design such a thick surface without adding worse aberrations.

(The next two articles describe how to design Schmidts and Maksutovs.)

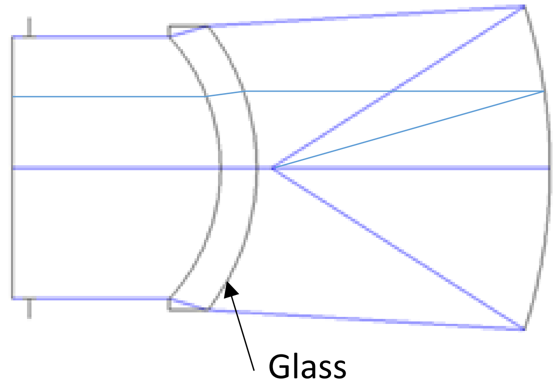

Figure 6. Maksutovs correct the image by filling the other corrector