EYEPIECE DESIGN by Dick Suiter

This article is about the design of eyepieces. By this, I don't mean intricate discussions about advantages of Nagler Types 3 vs. 4 or other such matters of interest only to specialists, but the reasoning that lead to the simplest forms of today. (Besides, I don't know the design of a Type 3 or 4!)

First of all, to design an eyepiece you have to understand what it is that you wish to do with telescopes in general. An eyepiece's chief job is to convert a focused beam to a parallel or near parallel beam. It is thus related to a telescope operated in reverse. The whole combination is called an afocal device, in that it takes parallel light in the front and spits it out parallel in the back. Another function is to get the all the light into the eye.

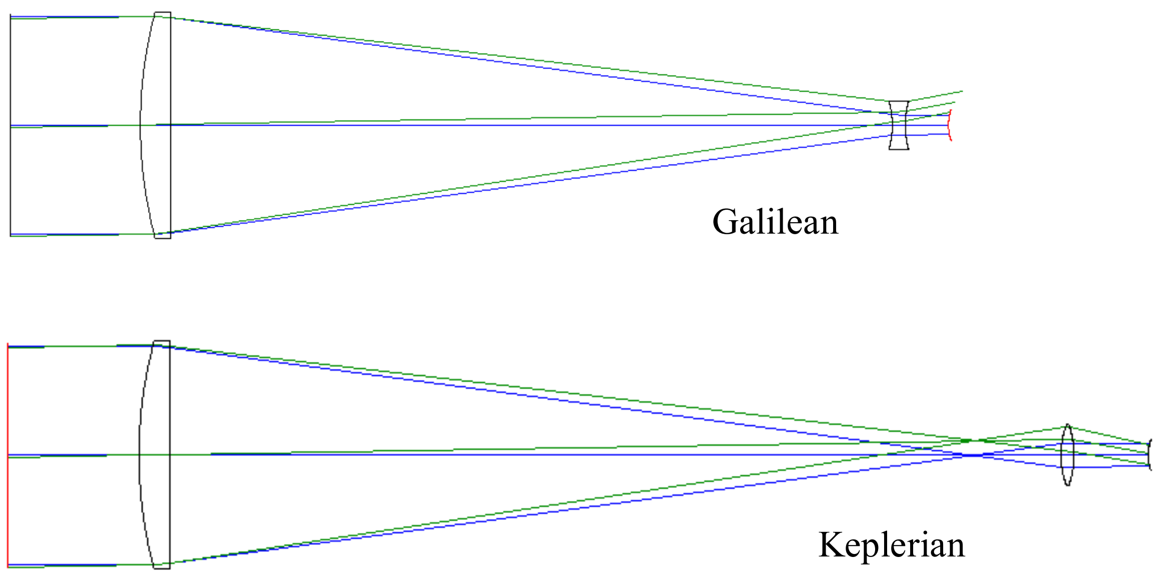

Figure 1. Two types of telescope/eyepiece systems making up a small 10-power spotting scope. The final curved surface is a 7 mm diameter human eye pupil. Galilean telescopes are inherently shorter but it is difficult to get the light into the eye, hence they are no longer used.

Here's the difference between a telescope and an eyepiece. A telescope deals with a very small beam, a degree or so in size. An eyepiece is forced to collimate light incident from half the apparent field-of-view (FOV) angle. A modern eyepiece might have an apparent FOV of 50 degrees and a half-angle of 25 degrees. Another difference is the position of the stop. The stop of the objective lens or telescope mirror is very near its location. The aperture stop of the eyepiece is still very near the primary and nowhere near the eyepiece. It is no wonder that eyepieces do not resemble primary lenses or mirrors: they have much different starting conditions.

There are a group of parameters that are associated with eyepieces, and I am sure everyone has heard of them:

1) Focal length
2) Apparent Field of View
3) Eye Relief.

There are some others, which may be less known or even unknown to most readers:

4) Distortion
5) Spherical aberration of exit pupil
6) Field curvature
7) Limiting f-number
8) Lateral color.

Now, the parameter "1) focal length" could be measure roughly, I suppose, if you held it up in paper-burning mode and measured the size of images at focus, but the way it is usually defined is indirectly in combination with a primary.

$$f_{eyepiece} = \frac{f_{primary}}{M} .$$

Here, $f_{primary}$ is the focal length of the objective lens or mirror and M the angular magnification of the system. Thus, if the magnification of the system is 50 and the focal length of the primary is 1250 mm, then the focal length of the eyepiece is 1250mm/50 = 25 mm. Angular magnification the tilt of the off-axis emerging beam compared with the beam incident on the front of the primary. An additional subtlety is that this is only for angles very little deviated from the center of the field. Most people use this calculation the other way, reading the eyepiece focal length from the barrel and calculating the magnification.

The parameter "2) apparent field " is the angular size of the field stop as viewed by the eyepiece. Now, a bit of explanation of field stops is in order. A field stop is a dark ring set at the position of focus of the eyepiece, and the telescope too if the image is in focus. It is meant to limit the field of view to angles where the image is acceptable and it is also a baffle. For most eyepieces, it is the ring you see when the eyepiece is inverted. A field stop is that sharp edge you see in the field when you look through the instrument. In spite of appearances, this sharp edge has nothing to do with the edge of the primary lens or mirror. That is, by definition, completely out of focus. It is a characteristic of the eyepiece alone, as can be verified by removing the eyepiece and just looking through it.

The real field-of-view would be the apparent field-of-view divided by the magnification in a perfect world, but the eyepiece distorts. You can measure the real field by finding a star near the equator and measure the time it takes to cross a centered field. Take the time in minutes and multiply by 3.989 to derive the real field in degrees.

Parameter "3) eye relief" takes some explanation. Everyone (sort of) knows that it is a measure of the distance between the last piece of glass and the eye, but not everyone knows its precise definition and what it originates from.

If you point the scope at a white wall and draw your eye back about a foot from the eyepiece, looking at the eyepiece rather than through it, you will see a lit circle hovering on the outside of it. You may think that the circle is a projection of the field stop, but you would be wrong. You are seeing a little image of the objective. Test this by putting it in a different f-number telescope. It changes in size.

When you are looking through the telescope properly, you automatically center this little lit circle in the iris of your eye. For the Galilean eyepiece in the first picture above, this lit circle is deep *inside* of the eyepiece. This is negative eye relief, and is one more reason the Galilean eyepiece was abandoned so quickly in favor of the Keplerian (the other is that the apparent field of view is so small).

The eye relief is the distance between the last piece of glass and the internal iris of the eye. It should be at least 10 mm for non-eyeglass wearers and 20 mm for eyeglass wearers. Unfortunately, it often is frittered away by poor barrel design. I have seen eyepieces with small enough eye relief lose what little they have by extensions of the barrel. The last surface is well within the eyepiece.

	5mm pupil	exit pupil 10	15	20
	40	12	16	20
	50	14	19	24
FOV	60	17	22	28
deg	70	19	26	33
	80	22	30	39
	90	25	35	45

Table 1: A five-mm exit pupil held at 10, 15, and 20mm from the last piece of glass will need to be transmitted by windows at least as large as the red values in mm.

For eyeglass wearers who cannot merely refocus (and that is about all of us), eye relief is of paramount importance. It does no good to possess an 80-degree eyepiece if you can only see the center 30 degrees of it.

Eye relief also drives the physical size of the eyepiece. The hole you look through gets larger if you move back away from it. In Table 1 we see that to view the edge of a 90 degree field of view with a 5 mm pupil, that last little window of glass is at least 45 mm in diameter. The innards of the eyepiece are even larger.

THE PARTS OF AN EYEPIECE

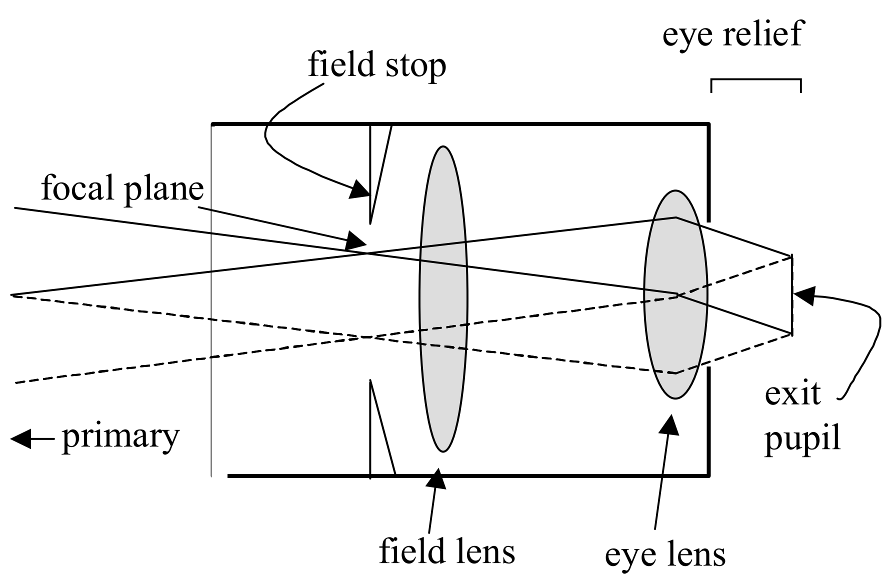

Figure 2. A sketch of the individual parts of a notional eyepiece.

The parts of a modern Keplerian eyepiece are shown in Figure 2. Note that there are two lenses, where most real eyepieces are pretty much filled with glass. These "lenses" represent the jobs of the various refractive surfaces. The refraction nearest the focal plane takes the place of the field lens and the refraction that takes place near the eye takes the place of the eye lens.

Most of the focusing power of the eyepiece is deposited in the eye lens. The field lens is placed so near the focal plane that it -- like a magnifying lens placed right against newsprint -- magnifies very little. So what is the field lens for?

It's simple. The field lens condenses light through the rear lens, like those old overhead projectors with the cheap Fresnel lens below the transparency surface and the little high-quality lens held well above the transparency surface. The cheap Fresnel lens did nothing for the resolution or the focus. It just squeezed the light through the upper lens. Otherwise the light goes everywhere and the image is dark. Here the situation is less severe, but the field lens prevents a falloff of light near the edge. Thus the field lens is *gathering light* that would otherwise miss the rear lens and the function of the eye lens is *magnification*.

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Part 2: ADVANCED CHARACTERISTICS

Let's define those other parameters of performance, namely, 4) distortion, 5) spherical aberration of exit pupil, 6) field curvature, 7) limiting f-number, and 8) lateral color.

4) *Distortion*. What you want is a rectilinear grid at the focal plane re-imaged as a rectilinear grid on the retina. What you get is considerably different, as is shown in Figure 3, which depicts the inner 33 degrees of a modified Kellner eyepiece.

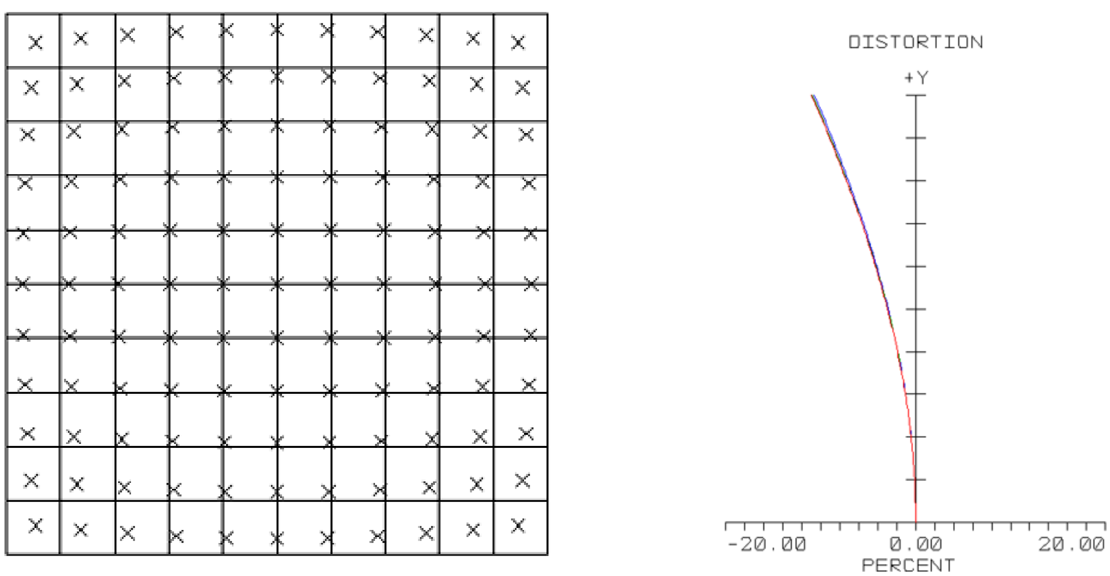

Figure 3. Barrel distortion in a modified König eyepiece.

A pattern pinched-in on the outside, or "barrel" distortion is much more common than the reverse "pincushion" distortion. One eyepiece was designed to reduce distortion, the so-called Abbe Orthoscopic eyepiece, and it reduces it to about 1/3rd of the value witnessed above for the modified König. For this, the designer had to add one piece of glass and form three pieces of glass into a difficult-to-make triplet.

5) *Spherical aberration of the exit pupil*. In the discussion above about eye relief, an assumption was made that all light goes through a little disk called the exit pupil. Your eye's iris, if placed right at the exit pupil, will then see all the light transmitted by the eyepiece. This assumption is not necessarily true. If the light is skewed off-axis, then a unique exit pupil is not a good approximation to what is happening. The way you tell if your eyepiece has spherical aberration of the exit pupil is to draw your eye behind it about a foot and locate it hovering above the last glass surface. It may help to brace a little toothpick or something against the eyepiece and place it right at the center of the exit. Then move your eye sideways and see if the exit pupil shifts sideways or seems stuck on the toothpick. If it shifts badly it either is exhibiting strong spherical aberration or the toothpick is placed at the wrong position. The effect of a significant shift is to possibly induce an effect called "kidney-beaning." If the eye is not placed just at the correct point, a bean shaped dark area shows up in the field of view. An eyepiece that showed this effect easily was the Nagler Type 1. (Actually, this eyepiece was never called a "Type 1," just a "Nagler," but when the Nagler Type 2 came out, this nomenclature became easy to extend.) The arrow of Fig. 4 shows the brown and red outer bundles (at 25 and 40 degrees apparent field radius) crossing at a point off the axis. The green bundle (10 degrees) has a far lower amount of side-shift compared with the blue bundle.

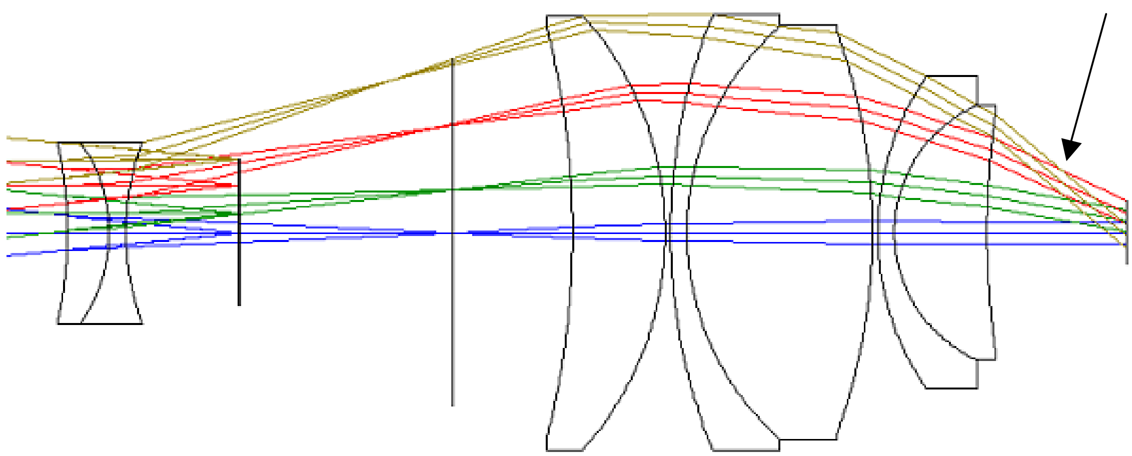

Figure 4. Spherical aberration of exit pupil in Nagler (Type 1) eyepiece (USP #4286844 scaled to 13mm)

Still, if you placed the eye accurately, even this eyepiece presented a good image. It seems that spherical aberration of the exit pupil is coupled with distortion, because the Orthoscopic eyepiece has practically none of it. The Ramsden disk (which is what the exit pupil is called) of an Abbe Orthoscopic is a true disk, an image of the objective hanging in space.

6) *Curvature of field*. Because we cannot always know what instrument the eyepiece will be fitted to, we must choose an eyepiece that fits the generic behavior of imaging a flat field. Optimally, the primary produces a flat field, which is then imaged accurately by the eyepiece. This is not always the case, but the best way of making a general-purpose eyepiece is to pretend that it is so. In the case of the modified König in Figure 5, we see a curvature away from the objective (to the left). In all cases of interest (normal or catadioptric Cassegrains, refractors, or

Newtonians), the curvature should go the other way. This curvature is not unusual, nor does this fact mean the eyepiece is particularly bad. It just means that the refocusing is placed on the natural accommodation power of the human eye. As bad as this curvature looks, it is corrected by only about 1/2 to 1 diopter of accommodation, which most people retain into their fifties.

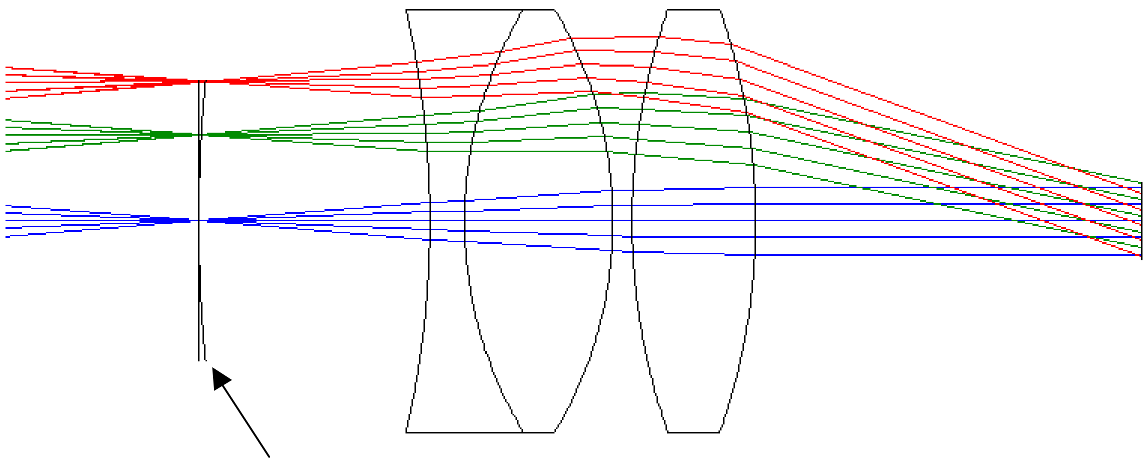

Figure 5. Backwards field curvature of modified König.

7) *Limiting f-number*. Depending on their complexity, eyepieces can process more (or less) steep light cones. Most are sharp near the center of the field, but they need a 10-degree field radius that is also approximately diffraction limited to provide a finite window in which objects can be seen, a "sweet spot" we might call it. Taking this as a criterion and searching on incoming light cones we see that, as a rule, more complicated designs will tolerate lower aperture ratios. Please note that the 13-mm Nagler type 1 was scaled up to 26 mm to make it more directly comparable. Such an eyepiece never existed. (The 13 mm tolerated *f*/5 pretty well.) Also note that I've included a non-optimized eyepiece as a simple sphere of glass, but this thick lens seems to simulate the action of a two-lens eyepiece because it is not as bad as a Kepler-type thin lens.

Table 2: Minimum f-number that yields a diffraction-limited field of 20 degree diameter (refocused)

	no. elements	f.l.	mimimum f-number
Nagler-1 scaled	7	26mm	*f*/9
Plössl	4	26mm	*f*/11
Abbe	4	28mm	*f*/11.5
modified König	3	29mm	*f*/12.5
Ramsden	2	25mm	*f*/20
sphere	1	25mm	*f*/25
Kepler simple lens	1	26mm	*f*/33

This table was generated by raising the focal ratio until the light in the 10-degree off-axis spot was approximately the same radius as the Airy disk. Both the Airy disk expands and the confusion spot diminishes in this viewpoint. The minimum f-numbers are probably far too conservative, but they are relatively the same. If you multiply an entry by a corrective factor, you would need to multiply the whole list. (If we divided the f-numbers by 2, we would get a more representative performance, I'm thinking.) Notice that the Kepler simple lens doesn't really do well. It seems that the long telescopes of the 17th century were not just long because of the simple-lens objectives. The eyepieces also weren't good.

The way this table was generated is depicted in Figure 6, which shows the performance of a Plössl at f/11 and f/5.

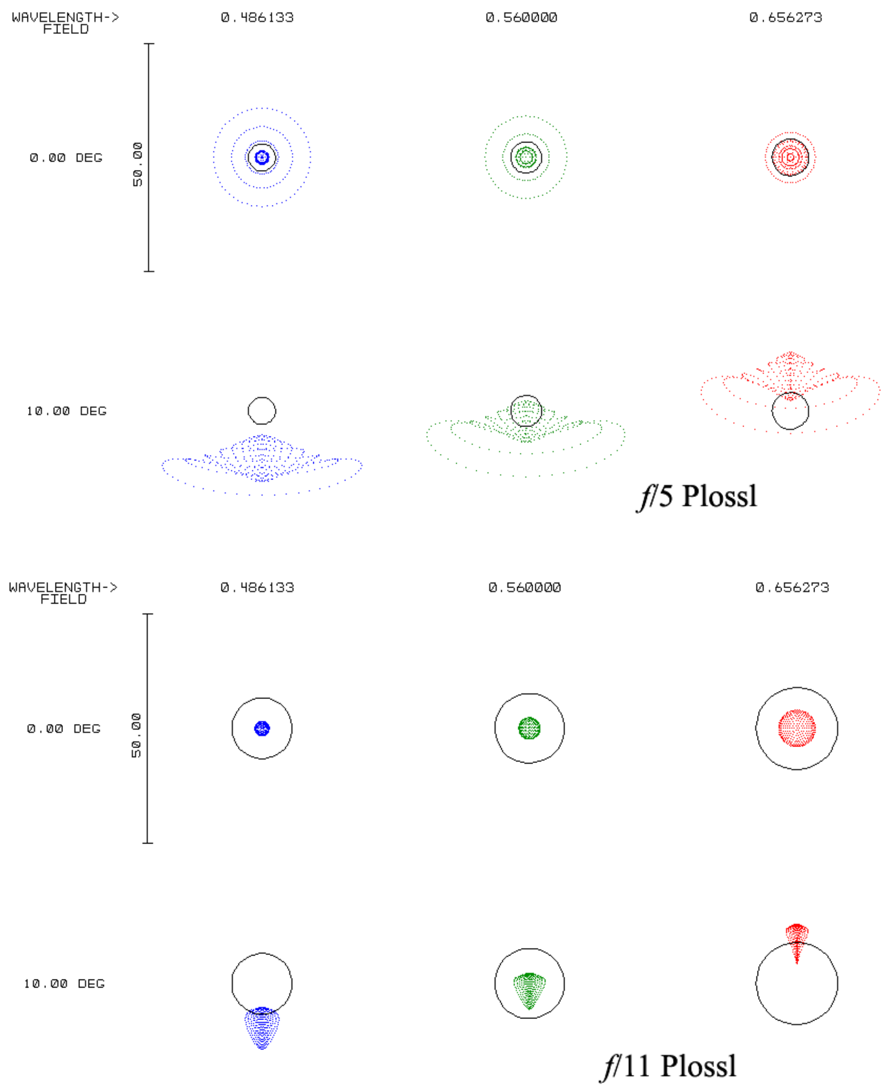

Figure 6. A picture showing the degradation at low f-number

An alternate calculation would be to demand 1 arcminute resolution. For this 26 mm eyepiece, that is

$$26\text{mm}/(57.3*60) = 0.00756 \text{ mm} = 7.5 \text{ micrometers}$$

which is somewhere between the f/11 and the f/5 performance.

8) *Lateral color*. Note in Fig. 6 the 10-degree spots don't hit the center of the Airy disk in the red and blue. That is lateral color or the diversion of the image into a little spectrum streak. It is caused by the thick lenses acting as prisms to off-axis light. Some lenses let the lateral chromatic

aberration go hang, without even attempting to correct it. But most at least try to draw the curve back on itself, as in Figure 7 on the right.

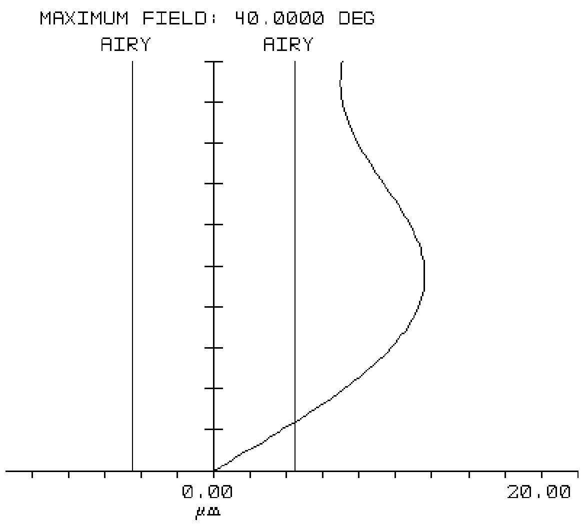

Figure 7. Well-corrected lateral color (Nagler 13, left), and adequately corrected lateral color (modified König 29 mm, right)

There was a one-glass corrective eyepiece called the Huygenian eyepiece that corrected lateral color (but not longitudinal). It is seldom used now because it works well only on long refractors or very slow reflectors. Its correction of lateral color is a somewhat theoretical victory because the off-axis coma is so much worse than lateral color.

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Part 3. More on eyepiece design will appear in the next installment!